

I.D. #	
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Subject	
Course	Phys. 560
Section	
Instructor	
Date	

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(Begin Writing on this page)

Lecture 14:

1.) Stark Ladder

We have been considering an electron in an electric field in the TBM. When

$$|\Psi(t)\rangle = \sum_n C_n(t) |n\rangle,$$

the E.O.M. for the amplitudes is

$$\dot{C}_n = - (C_{n+1} + C_{n-1}) - E_0 n C_n.$$

$$E_0 = e a \epsilon$$

we showed that

$$C_k = e^{-\frac{4it_0}{\epsilon} \sin \frac{E_0 t}{2k} \cos(-\frac{\pi k}{2} + ik)} \text{ and}$$

$$\langle n \rangle = 0$$

$$\langle n^2 \rangle = \left(\frac{4t_0 \sin \frac{E_0 t}{2k}}{\epsilon} \right)^2 \frac{1}{2\pi}$$

This is why the particles do not collect on one side of the lattice.

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They never move away from their initial site.

a) Eigenstates

$$C_m(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-ikm} e^{-4it\frac{\sin E_0 t}{E_0}} \cos(k - E_0 t/2).$$

$$\text{Let } k \rightarrow k + E_0 t/2.$$

$$\Rightarrow C_m(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-i((k + E_0 t/2)m)} e^{-4it\frac{\sin E_0 t}{E_0}} \cos k dk.$$

Note: There is no change in the limits of integration since we are integrating over the entire unit cell. Here is a useful identity:

$$e^{ix \cos k} = \sum_{n=-\infty}^{\infty} i^n J_n(x) e^{ink}.$$

J_n 's are Bessel functions.

$$\text{so } C_m(t) = e^{-i\frac{E_0 mt}{2}} \sum_{n=-\infty}^{\infty} i^n J_n\left(\frac{4t}{E_0} \sin \frac{E_0 t}{2}\right) \underbrace{\int_{-\pi}^{\pi} \frac{dk}{2\pi} e^{ink} e^{-ikm}}_{\delta_{nm}}$$

$$= i^m e^{-i\frac{E_0 mt}{2}} J_m\left(\frac{4t}{E_0} \sin \frac{E_0 t}{2}\right)$$

so we see the solutions involve Bessel functions. Here's the real reason why.

b.) Stark Ladder

$$E C_n = -n E_0 C_n - t_0 (C_{n+1} + C_{n-1}).$$

In general

$$E = \pm m E_0.$$

$$\Rightarrow \left[\frac{(n \pm m) E_0}{-t_0} \right] C_n = C_{n+1} + C_{n-1}.$$

Let's first determine the $E=0$ solution.

$$\frac{n E_0}{-t_0} C_n = C_{n+1} + C_{n-1}.$$

Recall Bessel's identity: $\frac{2n}{x} J_n = J_{n+1} + J_{n-1}$.

$$\text{Let } x = -\frac{2t_0}{E_0}.$$

$$\Rightarrow \frac{2n}{x} C_n = C_{n+1} + C_{n-1}.$$

$$\Rightarrow C_n = J_n \left(x = -\frac{2t_0}{E_0} \right).$$

Let's write the $E=0$ state in the following way

$$\langle E=0 | \Psi(t) \rangle_E = \sum_n C_n \langle E=0 | n \rangle$$

$$= \sum_n J_n \left(-\frac{2t_0}{E_0} \right) \langle E=0 | n \rangle.$$

What about $\pm E_0$?

$$(n \pm 1) \frac{E_0}{-t_0} C_n = C_{n+1} + C_{n-1}$$

$$2(n \pm 1) \frac{E_0}{-2t_0} J_{n \pm 1} \xrightarrow{\begin{array}{l} + \\ - \end{array}} J_{n+2} + J_n$$

$$J_n + J_{n-2}.$$

$$\Rightarrow \langle \pm E_0 | \psi(t) \rangle = \sum_n J_{n \pm 1} \left(\frac{-2t_0}{E_0} \right) \langle \pm E_0 | n \rangle.$$

in general

$$\psi_{\pm m} = \langle \pm m E_0 | \psi(t) \rangle = \sum_n J_{n \pm m} \left(\frac{-2t_0}{E_0} \right) \langle \pm m E_0 | n \rangle$$

The state with energy $\pm m$ has its maximum amplitude on site $n \pm m = 0$ or site $\pm m$. The state with energy E_0 is peaked at site -1 .

$\Rightarrow J_{\pm m}$ is the amplitude that a particle is on site n with energy $\pm m$. The $\psi_{\pm m}$'s are the Stark Ladder states. Note there is an $E=0$ state because the potential is unbounded. \Rightarrow there is no zero point energy. $\Delta x = \infty$ and $\Delta p = 0$.

2.) Momentum Relaxation

To get long-range transport, one needs to include some sort of mechanism for

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relaxation of the momentum.

$$\dot{P} = -eE - m\dot{r}$$

$$\text{and } \boxed{\dot{r} = \frac{\partial \Sigma}{\partial \lambda k}}$$



dissipation mechanism.

You will solve this as a HW problem.

One word on the convention for particles and holes.

$$\Sigma_h = \Sigma_0 - 2t \cos ka$$

$$V_k = \frac{2ta \sin ka}{k}$$

$$\frac{1}{m} = \frac{1}{k^2} \frac{2^2 \Sigma}{2t^2} = \frac{2ta^2}{k^2} \cos ka$$

$$\cos ka \geq 0 \quad k \in [0, \pi/a] \quad M \geq 0$$

$$\cos ka \leq 0 \quad k \in [\frac{\pi}{2a}, \pi/a] \quad M \leq 0$$